NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 6
COURSE CODE: NUM701S	**COURSE NAME:** NUMERICAL METHODS 1
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. All numerical results must be given using 5 decimals where necessary unless mentioned otherwise. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 2 PAGES (Including this front page)

Attachments

None

QUESTION 1 [30 Marks]

1.1. Given the function

$$f(x) = \frac{1}{1+x^3}$$

1.1.1 Find the third-degree Taylor polynomial for $f(x)$ about $x_0 = 0$ and use it to approximate $f(2)$. Find the error in this approximation and indicate if this is a good approximation. Justify your answer. **[13 pts]**

1.2. Newton's method applied to the equation $f(x) = x^3 - x = 0$ takes the form of the iteration

$$x_{k+1} = x_k - \frac{x_k^3 - x_k}{3x_k^2 - 1}, \quad k = 0, 1, 2, \ldots$$

1.2.1 Study the behaviour of the iteration when $x_0 > 1/\sqrt{3}$ to conclude that the sequence $\{x_k\}_{k\geq 0}$ approaches the same root as long as you choose $x_0 > 1/\sqrt{3}$. **[7 pts]**

1.2.2 Assume $-\alpha < x_0 < \alpha$. For what number α does the sequence always approach 0? **[5 pts]**

1.2.3 For an arbitrary $f(x)$, suppose that $f'(x)f''(x) \neq 0$ in an interval $[a, b]$, where $f''(x)$ is continuous and $f(a) \times f(b) < 0$. Show that if $f'(x)f''(x) > 0$, for $x_0 \in [a, b]$, then the sequence $\{x_k\}_{k\geq 0}$ generated by Newton's method converges `monotonically` to a root $\alpha \in [a, b]$. **[5 pts]**

QUESTION 2 [40 Marks]

2.1. Write down in details the formulae of the Lagrange and Newton's form of the polynomial that interpolates a function f at the set of data points $(x_0, f(x_0)), (x_1, f(x_1)), \ldots, (x_n, f(x_n))$. **[7 pts**

2.2. Use the results of the previous question to determine The Lagrange and Newton's forms of the polynomial that interpolates a function at the points $(0, 5)$, $(2, 15)$ and $(4, 41)$. **[18 pts**

2.3. If using the following formula to compute an approximation of $f'(x)$: **[15 pts**

$$f'(x) \approx \frac{1}{12\,h}\left[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)\right],$$

find the order of convergence as $h \to 0$.

QUESTION 3 [30 Marks]

3.1. Given the Initial-value problem (IVP)

$$y' = t + \frac{3y}{t}, \quad 1 \leq t \leq 2, \; y(1) = 0$$

3.1.1 Write down in details the second-order Runge-Kutta (RK2) algorithm to solve the specific IVP given above. **[5 pts]**

3.1.2 Compute an approximation to $y(2)$ after three iterations using the algorithm given in the previous question. **[10 pts]**

3.1.3 The exact solution to the above IVP is $y(t) = t^3 - t^2$. Show that the RK2 method is second-order accurate. **[15 pts]**

END OF PAPER
TOTAL MARKS: 100